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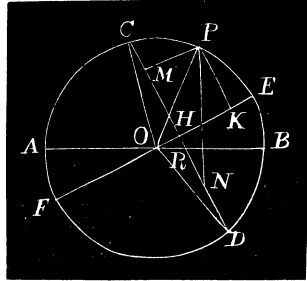
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330. "Two points are taken at random within a circle on opposite sides of a given diameter, and a third point is taken at random in the circumference; find the average area of the triangle formed by joining the points."

SOLUTION BY PROF. E. B. SEITZ, KIRKSVILLE, MO.

Let $ACBD$ be the given circle, AB the given diameter, M, N two random points on opposite sides of AB , CD the chord through them, P a random point in the circumference, and O the center of the circle. Draw the diameter EF perpendicular to CD , and PK perpendicular to EF .



Let $OA = r$, $RM = x$, $RN = y$, $RC = u$, $RD = v$, $\angle COH = \theta$, $BOH = \varphi$, and $POE = \psi$. Then we have

$$u = r \sin \theta + r \cos \theta \tan \varphi, \quad v = r \sin \theta - r \cos \theta \tan \varphi,$$

$$\text{area } MNP = \frac{1}{2}r(x+y)(\cos \psi - \cos \theta) = u_1, \text{ when } \psi < \theta,$$

$$\text{and } \text{area } MNP = \frac{1}{2}r(x+y)(\cos \theta - \cos \psi) = u_2, \text{ when } \varphi > \theta,$$

An element of surface at M is $r \sin \theta d\theta dx$, and at N it is $(x+y)d\varphi dy$, and an element of the circumference at P is $rd\psi$. The limits of θ are 0 and $\frac{1}{2}\pi$; of φ , $-\theta$ and θ , and doubled; of x , 0 and u ; of y , 0 and v ; and of ψ , 0 and θ , and θ and π .

By limiting P to the semi-circumference ECF , the whole number of ways the three points can be taken is $\frac{1}{2}\pi^2 r^4 \cdot \pi r$; hence the required average is

$$\begin{aligned} & \frac{8}{\pi^3 r^5} \int_0^{\frac{1}{2}\pi} \int_{-\theta}^{+\theta} \int_0^u \int_0^v \left\{ \int_0^\theta u_1 r d\psi + \int_\theta^\pi u_2 r d\psi \right\} r \sin \theta d\theta d\varphi dx (x+y) dy \\ &= \frac{4}{\pi^3 r^2} \int_0^{\frac{1}{2}\pi} \int_{-\theta}^{+\theta} \int_0^u \int_0^v (\pi - 2\theta + 2 \tan \theta) \sin \theta \cos \theta d\theta d\varphi dx (x+y)^2 dy \\ &= \frac{2r^2}{3\pi^3} \int_0^{\frac{1}{2}\pi} \int_{-\theta}^{+\theta} (\pi - 2\theta + 2 \tan \theta) (7 \tan^2 \theta + \tan^2 \varphi) (1 - \cos^2 \theta \sec^2 \varphi) \sin \theta \cos^3 \theta d\theta d\varphi \\ &= \frac{4r^2}{9\pi^3} \int_0^{\frac{1}{2}\pi} [(\pi - 2\theta) \cos \theta + 2 \sin \theta] (24 \sin^2 \theta - 3\theta + 3 \sin \theta \cos \theta - 22 \sin^3 \theta \cos \theta) \sin \theta d\theta \\ &= \frac{r^2}{\pi} \left[\frac{13}{18} + \frac{352}{81\pi^2} \right]. \end{aligned}$$

SOLUTION OF MISCEL. PROB. (2), P. 149, VOL. VII, BY PROF. SCHEFFER.

If in the series $S = ax + bx^2 + cx^3 + dx^4 + ex^5 + \dots$ we put $x = y \div (1 + vy)$, we get $S = ay(1 + vy)^{-1} + by^2(1 + vy)^{-2} + cy^3(1 + vy)^{-3} + dy^4(1 + vy)^{-4} + \dots$

Expanding by the Binomial Theorem, we obtain

$$ay - (av - b)y^2 + (av^2 - 2bv + c)y^3 - (av^3 - 3bv^2 + 3cv - d)y^4 + \dots$$

Substituting for y its equiv't $x \div (1 - vx)$, we have the req'd transformation.

[This problem was solved in a similar manner by Prof. Kershner.]